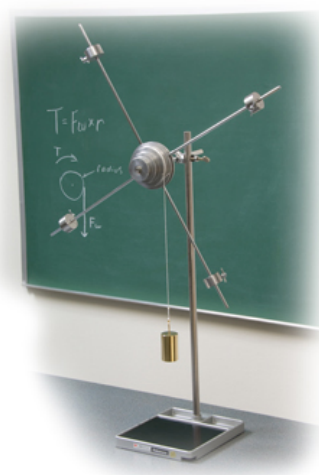# Rotational Inertia Demonstrator

P3-3545

BACKGROUND:

The Rotational Inertia Demonstrator provides an engaging way to
investigate many of the principles of angular motion and is intended for use high school physics
classes and university introductory physics courses. Whether demonstrating how changes in the
moment of inertia effect acceleration in a lecture or making quantitative measurements in the lab
the Rotational Inertia Demonstrator impressively accomplishes both. The apparatus easily attaches
to a ring stand. When clamped to the edge of a lab table, it will allow a falling mass to produce a
uniform angular acceleration. Hang a mass from a string in which one end is wrapped around one of
the three pulleys; this allows the mass to accelerate to the floor spinning the Rotational Inertia
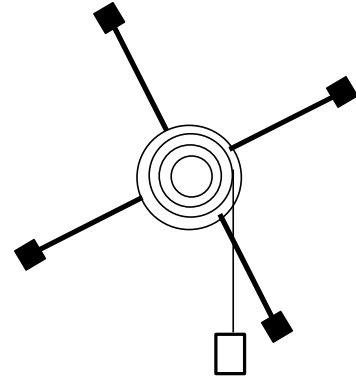Demonstrator above.

To investigate the effects of changes in torque and inertia, simply move the rope to a pulley of
different radius, change the amount of hanging mass, or move the masses on the spokes to change
the moment of inertia. Qualitative observations are a snap, and students can easily see the dramatic
effects of their adjustments. The apparatus lends itself to accurate quantitative experimentation as
well, with two ways of finding the moment of inertia: theoretically, according to the component
masses and their dimensions, and experimentally, by measuring the angular acceleration for a given
torque.

REQUIRED EQUIPMENT:

Rotational Inertia Demonstrator
Stop watch
Ring Stand
Clamp Holder
C Clamp
Hooked Mass set

EXPERIMENTS IN ROTATIONAL MOTION:

In three experiments you will study kinematics and dynamics of rotational motion using the Rotational Inertia Demonstrator. In the first experiment, angular acceleration will be determined by two methods and their values compared. Likewise, in the second experiment, comparisons of the calculated value of the moment of inertia by two methods will be made. In the final experiment you will apply the law of conservation of energy. Since there is very little friction, a good approximation of the conservation of mechanical energy can be obtained.

INSTRUCTIONS FOR ASSEMBLY:

When you receive your Rotation Inertia demonstrator, you will need to assembly the radial arms and the pulley gear. Securely tighten the radial arms in the pulley gear's radial arm screw sockets. The Teflon Tape will allow for a snug fit of the rods coarse threads in the socket and prevents unwanted vibrations during precision rotation experiments. Make sure all weights are securely tightened to each radial arm before starting experiments.

When securing the apparatus to the ring stand, the clamp holder (S-clamp) should be placed as close to the rotating wheel as possible.

TEACHER NOTES:

EXPERIMENT #1 KINEMATICS OF ROTATIONAL MOTION

Procedure: (Method 1) Calculating the Angular acceleration from the total time and the angular displacement (θ); using the equation [$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$] where $\omega_i = 0$:

There are two ways to measure the angular displacement (θ).

Start by hanging a mass from a string in which one end is wrapped around one of the three pulleys. Allow the mass to descend until it just touches the floor. Now raise the mass by winding up apparatus until it has rotated a complete number times (4 to 6 times). When released the falling mass will accelerate the apparatus resulting in an angular displacement equal to the number of rotations used to raise the mass. The angular displacement (θ) in radians can be found by multiplying the number of rotations by 2π. (Five rotations times $2\pi = 31.4$ rad.)

A more accurate value is obtained by dividing the radius of the chosen pulley into the vertical displacement of the falling mass. In the sample calculation below, the hanging mass is raised to a height of 1.0 m. By dividing the length of the string by the radius of the pulley, the number of radians of rotation is obtained. In this example a pulley radius of 0.0395 m is divided into a vertical decent of 1.00 m. This gives an angular displacement of 25.3 rad. With the displacement and a time to fall of 8.04 s the angular acceleration can be calculated.

Finding the angular acceleration of the falling mass:

Equation: Calculating (α) $\alpha = 2\,\theta/t^2$

Substitution: $\alpha = 2(25.3)/(8.04)^2$

Answer $\alpha = $ _____0.783 rad/s^2

(Method 2) In the second method, angular acceleration is determined from the linear acceleration and the radius of the pulley.

Finding the linear acceleration of the falling mass:

Vertical displacement 's' = ___1.00_____ m

Equation: Calculating (a) $a = 2s/t^2$

Substitution: $a = 2(1.00)/(8.04)^2 = 0.0309$ m/s^2

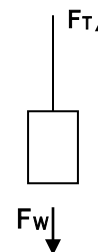**Transforming linear acceleration into angular acceleration:**

Pulley radius ___0.0395___ m

Answer $\alpha = a/r = .0309/.0395 = 0.783$ rad/s^2

As can be seen from the sample data, close agreement between the two methods is expected.

EXPERIMENT #2 DETERMINING THE MOMENT OF INERTIA

(**Method 1**) In this experiment the rotational inertia 'I' of the apparatus is found by applying Newton's second law for rotational motion: ($\tau = I \alpha$). Here the torque 'τ' is produced by the tension in the string 'F_T'. To calculate this tension, the mass of the falling weight and its vertical acceleration from experiment 1 is needed. Using the equation $F_W - F_T = ma$ the value (F_T) can be determined.

$F_T = F_W - ma = (.20)(9.81) - (.20)(.0309) = 1.956$ N

Combining the value of (a) the tension in the string 'F_T', (b) the radius of pulley 'r' and (c) the angular acceleration of the apparatus 'α' from above, we can solve for 'I' the moment of inertia using ($\tau = I \alpha$), where $\tau = F_T$ x r

Equation: Calculating (I) $I = F_T$ x r/ α

Substitution: $I = (1.956$ N$)(0.0395$ m$)/ (0.783$ rad/s$^2)$

Answer $I = $ _.0988 kg m^2

(**Method 2**) Direct Determination of Moment of Inertia from formulas. When adding up the moment of inertia of each of the parts of the apparatus, the value of the pulley component of the apparatus cannot be easily determined. This value is therefore provided for the students and is given to be (.00058 kg m^2). Sample data is given below based on the moveable masses being placed at the far end of the rods.

I(pulleys) _____.00058 _____ kg m^2 (provided by teacher)

Thin Rods: length _____ m,   mass _____ kg

I (thin rods) _____ kg m^2 (times four) ____0.0127___ kg m^2

Movable mass: mass _____ kg   distance from center _____ m

I (movable mass) _____ (times four) ____0.0834___ kg m^2

Σ I = I (pulleys) + I (rods) + I (movable masses) = ___.0968_ kg m^2

EXPERIMENT #3 ENERGY OF ROTATIONAL MOTION

Using the equations for translational and rotational kinetic energy and comparing their total gained to the loss of gravitational potential energy of the system, a good approximation of the conservation of mechanical energy can be found.

$\Delta KE(total) = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2$ $\Delta PE = mg\Delta h$

I (kg m^2)	ω_f (rad/s)	KE(rot) (J)	KE(tran) (J)	KE (total) (J)	ΔPE (J)
0.0988	6.294	1.957	0.00619	1.963	1.962

Above is the result of sample calculations from the lab representing good confirmation of the conservation of mechanical energy.

STUDENT LABS:

EXPERIMENT #1 KINEMATICS OF ROTATIONAL MOTION

As any object starts spinning faster and faster it is experiencing a type of acceleration called angular acceleration (α). In this experiment with the Rotational Inertia Demonstrator you will calculate this acceleration using two different methods and compare their results.

Procedure:

(Method 1) Calculating angular acceleration (α) using $[\theta = \omega_i t + \frac{1}{2} \alpha t^2]$ where $\omega_i = 0$:

1. Start by hanging a mass from a string in which one end is wrapped around one of the three pulleys. Rotate the apparatus allowing the mass to descend until it just touches the floor. Now raise the mass by winding up apparatus until it has rotated a complete number of times (4 to 6 times). Measure the vertical distance the hanging mass was raised above the floor 's' and record this distance in the space provided in method 2. When released, the falling mass will accelerate the apparatus resulting in an angular displacement equal to the number of rotations used to raise the mass. This angular displacement (θ) can be found by multiplying the number of rotations by 2 π.

> # Rotations _____ = _____ radians

Timing the free fall:

2. Start with the all moveable masses on the rods placed at the same distance from the center. Raise the hanging mass back up to its initial distance above the floor and time its fall to the floor as it freely accelerates the apparatus rest. Repeat this process of measuring the time to fall, until you have at least three consistent values. Average these three values.

> (t_1) _____ s (t_2) _____ s (t_3) _____ s (t_{avg}) _____ s

Calculating the Angular acceleration:

3. To calculate this acceleration you will use three different values from your experiment. First, since the falling weight and the apparatus were initially at rest, the starting angular velocity was zero (ω_i). Second, the total angular displacement in radians (θ) was determined from the number of rotations the apparatus made.

    Knowing the average time of acceleration, the angular displacement and the initial angular velocity, calculate the average angular acceleration.

Finding the angular acceleration of the falling mass:

Equation: Calculating (α) α =

Substitution: α =

Answer α = _____ rad/s^2

(Method 2) calculating angular acceleration using ($a = \alpha r$)

4. First, calculate the linear acceleration (a) of the falling mass from the data above. Use the vertical displacement (s), time to fall and the initial velocity (v_i) of the mass, which was zero, to make this calculation.

Vertical displacement 's' = _____ m

Equation: Calculating (a)   a =

Substitution: a =

Answer a = _____ m/s^2

5. Determine the radius of the pulley, which the string is wrapped around, by measuring its diameter with a caliper of ruler. Points on the pulley at the radius where the string is unwinding, experience a tangential acceleration (a_T), which is the same as the linear acceleration of the falling mass.

   Pulley radius _____ m

6. Use the relationship between the angular acceleration (α) of a rotating object and the tangential acceleration (a_T) of a point on the object at a given radius (r), to determine the angular acceleration of the apparatus.

Equation: α =

Answer α = _____ rad/s^2

7. Compare the two acceleration values and determine if they agree within the uncertainty of measurement.

EXPERIMENT #2 DETERMINING THE MOMENT OF INERTIA OF THE ROTATIONAL INERTIA DEMONSTRATOR

When an unbalanced torque is applied to a body it will experience an angular acceleration in accord with Newton's second law. Here the net force becomes the net torque, and mass becomes the moment of inertia. Calculating the angular acceleration as in experiment 1 and using the torque produce by the tension in the string connected to the falling mass, you will calculate apparatus's moment of inertia. Next, you will calculate the moment of inertia a second time by the direct application of the formulas for the moment of inertia of geometric objects.

Procedure:

(Method 1) Using Newton's 2nd Law

1. This method will require you to use the calculated values from experiment 1 for both the vertical acceleration of the falling mass and the angular acceleration of the rotating apparatus. In addition, you will need to use the same value of the falling mass and the radius of the pulley used in the experiment. Record these acceleration values below.

    Angular acceleration of rotating apparatus (Exp.1, Method 1) _____ rad/s^2

    Linear Acceleration of falling mass (Exp.1, Method 2) _____ m/s^2

2. Below you will calculate the torque produced by the string attached to the falling mass. This value is a product of the tension (F_T) in the string and the radius of the pulley you chose to wrap the string around. Record the radius of the pulley used in experiment 1.

    Radius of pulley _____ m

3. To calculate the tension in the string, use the value of the linear acceleration of the falling mass and its weight. Here, the tension of the string pulling down on the side of the pulley is the same tension pulling up on the falling mass. What two forces are acting on the falling mass and which is greater? Draw a free body diagram of these forces and use Newton's second law to calculate the string tension.

    Falling mass (m)_____ kg

    Weight of falling mass (F_W)_____ N

Equation: Calculating (F_T) F_T =

Substitution: F_T =

Answer F_T = _____ N

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4. Now that you have the value of (a) the tension in the string, (b) the radius of pulley and (c) the angular acceleration of the apparatus, solve for 'I' the moment of inertia using Newton's Law $(\tau = I\alpha)$.

Equation: Calculating (I)   I =

Substitution: I =

Answer I = _____ kg m^2

(Method 2) Direct Determination of Rotational Inertia from formulas

The direct method requires you to find the total moment of inertia by adding up the moment of inertia of each of the parts of the apparatus. To do this, your will need to know the masses and dimensions of the various parts of the system. Since all the thin rods and moveable masses are the same you only need to measure one of each. The moveable masses on the rods are treated as point-masses and have the same formula for moment of inertia as a hoop. Unscrew one of the thin rods and find and record its mass. For its length, use the distance from the center of the axis of rotation to its end point. Even though this ignores the short distance near the center, you will still have a good approximation. Finally, the pulleys assembly can be treated as and series of disks, however, since these cannot be individually measured, the moment of inertia of the pulley assembly will be provided by the lab instructor. Remember, that the moment of inertial of a system is just the sum of the moments of inertia of the individual parts.

I(pulleys) _____ kg m^2 (provided by teacher)

Thin Rods: length _____ m,   mass _____ kg

I(thin rods) _____ kg m^2 (times four) _____ kg m^2

Movable mass: mass _____ kg   distance from center _____ m

I (movable mass) _____ (times four) _____ kg m^2

Σ I = I (pulleys) + I (rods) + I (movable masses)

Σ I = _____ + _____ + _____ = _____ kg m^2

Compare the moment of inertia values from methods 1 & 2 and determine if they agree within the uncertainty of measurement.

EXPERIMENT 3   ENERGY OF ROTATIONAL MOTION

Like the energy that runs a Grandfather clock, the Rotational Inertia Demonstrator is powered by the potential energy of a falling weight. With friction extremely small, the transfer of energy within the system represents a good approximation of the conservation of mechanical energy. As the falling hanging mass loses gravitational potential energy, it and the rotating apparatus above gain kinetic energy. The purpose of this activity is to calculate and compare the final kinetic energy of the system to the loss of potential energy of the falling mass.

Determining the initial mechanical energy of the system.

1. With the apparatus initially at rest and the hanging mass raised above the floor, the total mechanical energy of the system equals the potential energy of the hanging mass. Measure and record its mass and height through which is will fall.

Gravitational Potential Energy

Falling mass:

Mass _____ kg   Vertical displacement _____ m

Equation: Calculating (Potential Energy PE)   PE =

Substitution: PE =

Answer PE = _____ J

Translational Kinetic Energy

2. To find the kinetic energy of the falling mass as it strikes the floor, you will need to know its final velocity. As in experiment 1, raise the falling mass to its initial position and time its descent to the floor, repeating this until you have three consistent values. Record their average.

(t_1) _____ s (t_2) _____ s (t_3) _____ s (t_{avg}) _____ s (v_{avg}) _____m/s

3. Next, use the vertical displacement (s), the average time to fall and the initial velocity (vi) of the mass (which was zero), to calculate the average velocity with which the mass strikes the floor. From the mean speed equation we know that velocity final is two times the average velocity. With this value, continue and calculate the falling mass's kinetic energy as it reaches the floor.

Equation: Calculating Kinetic Energy   KE =

Substitution: KE =

Answer KE_f = _____ J

Rotational Kinetic Energy

4. The equation for rotational kinetic energy has the same format as translational kinetic energy. In the rotational equation, the mass is replaced with its moment of inertia (I) and the final velocity is replaced with its final angular velocity (ω_f). . Use the value for the moment of inertia you calculated in experiment 2. V_f for the falling mass is the same tangential velocity of the pulley with the string wrapped around. From the equation ($v = r \omega$), the value of ω_f will be calculated by dividing above V_f for the falling mass by the radius of the pulley.

Radius of pulley _____ m ω_f = _____ rad /s   I = _____ kg m^2

Equation: Calculating Rotational Kinetic Energy KE =

Substitution: KE =

Answer KE = _____ J

KE(total) = _____(tran.) + _____ (rot.) = _____ J

5. Compare below the sum of the two final kinetic energies to the loss of potential energy of the falling mass and determine if they agree within the uncertainty of measurement.